Learning Number Facts

a guide to the principles and practice of number fact learning

a whitepaper by Flurrish Education | www.FlurrishEd.co.uk
in collaboration with Caddington Village School

Foreword

Schools that are successful for all know their pupils well – as learners. They constantly enquire into their knowledge levels, their proficiency in a range of skills and into how they are progressing. As a result, they understand what their pupils know now, what they need to know next – and how they learn. For children at all levels, enquiry driven schools are best placed to understand, guide and encourage them as they progress in their learning; keeping them on task, accelerating them when they are motivated and motivating them when they need help. Making children aware of the joy of learning and how to succeed in learning is of course powerful both for the individual and ultimately for society.

In this whitepaper Flurrish outline current understanding of how human memory works in relation to a vital aspect of learning – the understanding and recall of information. They go on to show how this informs the way many primary schools currently teach acquisition of number facts: those vital chunks of mathematical information that it is so important to access in an instant. However it is the link they make to more recent developments in optimising such learning using widely available technology that is so powerful here.

Teachers who understand how to use these linkages can begin to develop strategies in the classroom and at home for increasing mathematical knowledge and understanding. Examples of such approaches are provided along with evidence of their success on large samples of real children learning in real classrooms. They demonstrate the impact of using such embedded technology to enable massed practice, spaced repetition and feedback in the acquisition of number facts. You as reader can judge the impact yourself.

Education is constantly responding to the demands of the 21st century, so it is incumbent on all educators to enquire into, seek out and harness such approaches that can help our schools help our children to meet these demands.

Peter Dudley, Education Leader and Writer

About

Why this whitepaper?

Number facts (and particularly times tables) are one of the "essentials" for succeeding in life. We learn them from when we start school as they underpin the way we understand mathematics and progress in the subject. As adults, we use them daily for calculating shopping totals, estimating building quantities or picking the best unit costs. Memorising number facts allows us to focus our mental energy on the core task and not get bogged down in calculation. Yet many children, and too many adults, don't "know" their number facts - they struggle with recall, have difficulty with calculation and immediately place themselves at a disadvantage in daily life. A numerate (and literate) society is one we should strive for in order to reduce social disadvantage.

Memory controls encoding, storage and retrieval of information. Short term memory can hold up to about seven items, but these are rapidly forgotten. Long term memory – that is, the explicit (conscious) recall of information – is where number facts need to be stored. Current classroom techniques are founded upon good memorisation principles based around repetitive practise (e.g. chanting, whiteboards, rhyming), but don't scale well to massed practise for large groups. The Flurrish Number.fy app uses the best memorisation methods to maximize learning in the 21st century classroom: handheld, massed practise, rapid feedback, interleaved questions, standardized testing and the centralized recording/marking of questions. In trials, 65% of age 7 (Year 2) children knew their 12x tables after 15-minutes a day of practise in-class.

With thanks...

We are grateful to Sue Teague (Executive Headteacher), along with her pupils and staff at Caddington and Slip End schools, for playing so many games, answering so many questions, and being so willing to learn number facts. We would like to acknowledge the contributions of Sue Teague and Dr Tom Mitchell-Gallagher (Liverpool-Hope University) to this whitepaper.

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Introduction

Education

Education, that is **the knowledge or skill obtained or developed by a learning process** (thefreedictionary.com), happens as soon we are born. We learn to live and develop the aptitudes and capabilities to sustain us in our life. However it is *formal* education, a pre-requisite of any developed economy, where we see a prescribed curriculum of activities which directs our learning and has a significant impact upon future opportunities in life. With the transition of Western economies from manufacturing to knowledge-based (for example creative, design, technology and service industries), there has been an increasing focus upon "education" to enable children, the country's future workers, to be prepared for a life of work that will be dramatically different to that of their parents and grand-parents. The info-video Shift-Happens (YouTube) is a good example of this meme, highlighting the dramatic changes in the business world that are having increasingly rapid impacts upon society. Individuals wanting to live a full and meaningful life, must be prepared and ready for this challenge. In order to do so, education must develop the competencies that will be required, something John White calls "a properly rounded academic education" or, more simply, **autonomy**. Donald Clark, in reviewing this, noted that

the curriculum [...] needs to foster moral, intellectual, financial and practical autonomy to allow people to lead happy, healthy, lives, form relationships, cook, find jobs and think for themselves.

Within a national context, for example, the UK Government specifies the National Curriculum which is intended to:

promote the spiritual, moral, cultural, mental and physical development of pupils at the school and of society

prepare pupils at the school for the opportunities, responsibilities and experiences of later life

This provides a broad, prescribed, programme of learning of what the government believes is essential for future citizens. It is split into four Key Stages with learning including both knowledge (e.g. historical events) and skills (e.g. reading) across twelve areas of study, of which numeracy is one area. The National Curriculum specifies detailed expectations across each subject area, by level of study.

Rationale

Achievement in numeracy is a key aspect of modern life – yes, you need to be numerate for exam performance, but it is critical for career development and ultimately, life chances. This has been highlighted by the "Make Britain Count" campaign in The Telegraph and the charity National Numeracy. Developments that are able to boost ability in numeracy will increase self-esteem and confidence and have positive impacts across other aspects of life, including school attendance, employment, earnings and health. Ability in numeracy (just like literacy) is an *educational right* that all children should have. Whilst low achievement disproportionately affects those that are socially disadvantaged, boosting achievement would significantly improve both an individual's prospects as well as the national competitiveness of the workforce.

Key to both National Numeracy and "Make Britain Count" is evidence of low achievement and aspirations in numeracy both by children and adults, as well as the declining take up of numerate qualifications at A-level and beyond. However, what evidence exists for poor number fact knowledge that might contribute to this perceived malaise? Published research (Smith and Teague, 2014) for a cohort of 232 children aged 9-13 showed that only 80% of times tables questions were answered correctly, with the hardest questions having error rates of 65%. This was undertaken over a 2-week period, answering nearly 60,000 questions, using randomised sets of 20 times table questions (up to 12x). In short, the scale of the lack of number fact knowledge is much greater than might be anticipated so hindering the progress of children in understanding mathematical concepts and applying them to problem solving. Indeed, you *"don't know what you don't know"* and a lack of appropriate baseline information for teachers to measure progress is potentially slowing the development of foundational mathematical knowledge.

In addition, since the abolition of National Attainment Levels in England, schools have been required to develop their own metrics for monitoring achievement and progress (against the National Curriculum) both so they can use this information to drive improved standards and so that this can then be evidenced to external authorities.

It's within this context that this whitepaper begins (in Section 2) by outlining current knowledge and thinking about memory and how it works. Section 3 then presents a triplicate approach of **what**, **how** and **if; what** is it we need to achieve in number fact learning in schools, **how** is this undertaken and **if** we could design a programme of study to do this efficiently and effectively, what would it look like? Section 4 then introduces an example of how such a system might operate and the results from trials in several schools in the United Kingdom. We hope you enjoy the journey we take over the next few pages as it’s exciting and the future for disruptive change may be profound.

Case Study: Governing Body

By Helen Houldcroft, Chair of Governors, Caddington Village School

Following the reform of assessment practice in 2015 when 'life after levels' began, our Governing Body felt that we needed to accurately assess some of the fundamental aspects of learning that are pertinent to all primary school children.

We felt that the information presented must be understandable to non-professionals, of sufficient detail and produced regularly so that any gaps in progress could be identified. We decided to report key data on a half termly basis that would clearly and accurately show pupils reading and spelling ages, writing ability and knowledge of times tables/number bonds. This data has some limitations in that it does not show a pupil's ability to apply the knowledge and skills being acquired; however, it is reliable, accurate and gives us a firm basis for comparison.

Other data was also produced – teacher assessments for Reading, Writing, Grammar, Punctuation & Spelling, as well as for maths. Some very productive conversations could then take place linking the two datasets together – for example, how do reading ages compare to pupil attainment recorded by a teacher's assessment?

Whilst reading and spelling age tests have been around for decades (e.g. Vernon spelling since 1977) it would be fair to say that systematic testing of times tables and number bonds is not common, and rarely reported to Governors.

The investment in the Number.fy app in our school meant that we could fairly easily pull data together and demonstrate knowledge and progress of pupils regarding times tables and number bonds (see figure). This data cannot represent the breadth of the numeracy curriculum, but it does augment the other information we review and provides useful validation.

		100%		90% +		80-90%		<80%	
Year	# Pupils	#	%	#	%	#	%	#	%
1	49	33	67.3%	6	12.3%	1	2.0%	9	18.4%
2	56	1	1.8%	2	3.6%	7	12.5%	46	82.1%
3	55	1	1.8%	6	10.9%	8	14.5%	40	72.7%
4	40	13	32.5%	7	17.5%	2	5.0%	18	45.0%
5	42	10	23.8%	18	42.8%	7	16.6%	7	16.6%
6	61	21	34.4%	20	32.8%	9	14.8%	11	18.0%

Memory

How we remember

Memory is one of the core functions of the brain and is a critical element in all learning as it controls the **encoding**, **storage** and **retrieval** of information (Figure 1). Through *encoding* we take stimuli from the external world (sight, sound, smell, taste, touch) and create a more stable record that can be *stored* in memory for a period of time. This, of course, is only useful to us if we can *retrieve* the information for use at a later stage (Atkinson and Shiffrin, 1968).

Short-term memory is analogous to writing in the sand – it allows us to retain a list of information for a short period of time, usually seconds to minutes before a new tide of sensory information floods into our minds and memory rapidly fades. Research suggests the number of items we can store in short-term memory is about seven (Miller, 1956), one of the reasons why local telephone numbers were fixed at this length, although it's suggested that this may actually be closer to four (Cowan, 2000).

Long-term memory is more nuanced and complex, something we can experience simply by trying to remember: (1) the house we lived in as a child, (2) a memorable exam, (3) where we were when Princess Diana died or (4) the number registration of our first car. For example, a short steep drive (Nissan Sunny parked there) to a 1970s semi-detached brick house with long narrow hall from the front-door leading to the kitchen, knowing that 15g of gold is mined from 1 ton of rock in New Zealand, driving to see Granny on the motorway and HLX307V.

Long-term memory is commonly divided into two types: explicit and implicit. **Explicit** (or declarative) memory can be thought of as the conscious recall of information and is stored in two main ways. **Semantic** memory is concerned with abstract information about the world around us – facts, such as pi being 3.1415926. **Episodic** memory stores strong personal memories that can appear as fresh and new as when they were first formed – they typically encompass our senses and emotions and can often be triggered by similar events that makes their recall particularly vivid. For example, the death of a parent or the smell of washing on a Saturday morning. Visual memories can be particularly strong and recalled in detail.

Implicit (or procedural) memory concerns unconscious recall and is best exemplified through motor skills ("muscle memory") that are improved with continued practise. A tennis forehand drive to the baseline or the curving football freekick that arcs over the defence and arrows into the corner of the net – both perfected through hours of repetitive practise. Rote learning falls in this category – remembering your phone number or number plate. Within education, rote learning is used to memorise foundational information that form the building blocks of subject learning – in numeracy, number facts are commonly memorised.

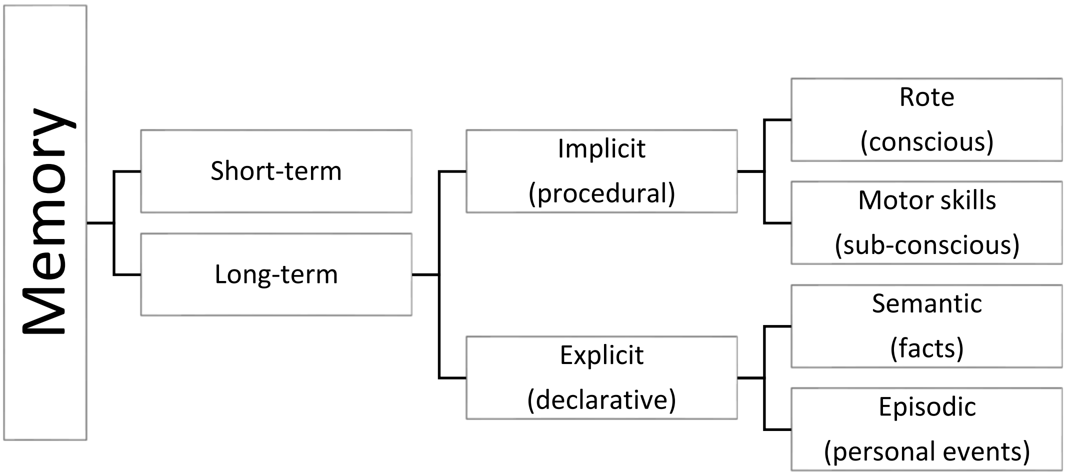

Figure 1 Types of memory

When we forget

We forget things; it's a fact of life. That extra item of shopping, your niece's birthday, the author of the last book we read, our bank account number. But now that we know *how* we remember, it's easier to understand that some of this information is stored in short-term memory and hasn't made it to long-term memory. Or, if it *was* in long-term memory, we can no longer easily recall it. Perhaps surprisingly, experimental understanding about forgetting goes back to the late-1800s and the research of Hermann Ebbinghaus. His most famous experiment involved remembering nonsensical syllables (they were entirely abstract). He tracked how long they could be remembered for – and the results were startling. After only **20 minutes** over 40% of information had been forgotten. By **1 day** 66% had been forgotten and by **1 month** nearly 80%! We can create a graph using this data (Figure 2) which is known as the **Forgetting Curve**.

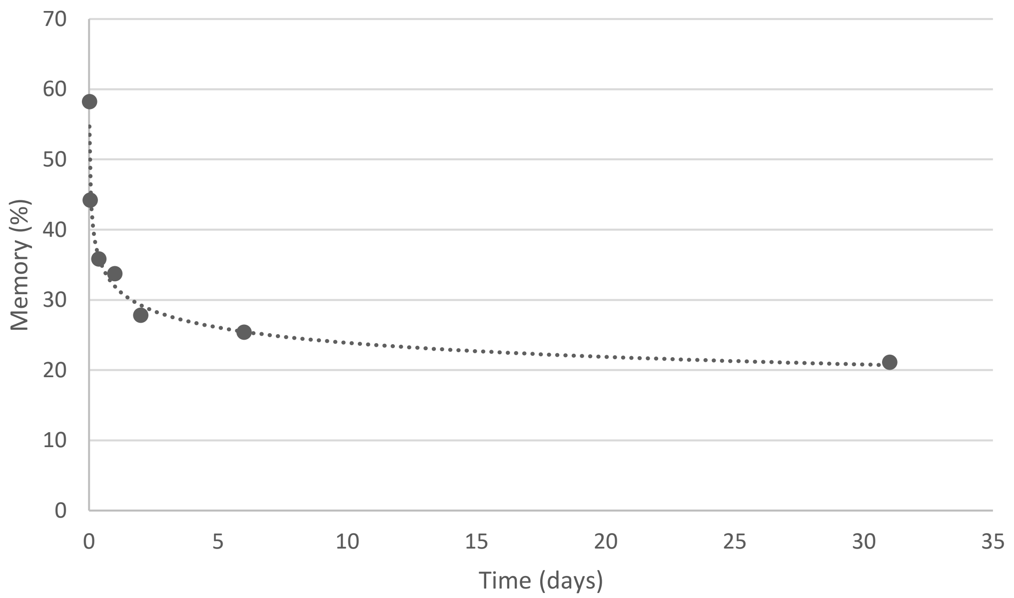

Figure 2 The Forgetting Curve (after Ebbinghaus)

The important result from this was that most knowledge is lost rapidly – very rapidly – and clearly demonstrated the difference between short-term and long-term memory. Learning needs to shift information into long-term memory to be effective and has profound implications for designing learning strategies. It is also strong evidence to demonstrate that *cramming* for exams has limited impact and is not an effective use of time (Kornell, 2009).

Learning Number Facts in School

This section presents a trifold approach to understanding the learning of number facts in school. When we say number facts, we mean both **number bonds** (that deal with the concepts of addition and subtraction) and **times tables** (that deal with the concepts of multiplication and division). Specifically, we focus on **what** it is children need to learn, both in terms of subject benchmarks, but also as prescribed by the UK government. We then move on to broadly outline **how** this is typically achieved using standard classroom methods. Finally, we outline what a learning system might look like **if** we took the current understanding on how memory operates that we presented in Section 2 - how could a programme of study do this efficiently and effectively.

WHAT we learn

Subject Benchmarks

Mathematics is a language and, in order to become fluent in it, children need to understand the underlying syntax. This begins with arithmetic, becoming aware of the notion of the number line (and real numbers), before looking at rational quantities and how they are represented through the decimal (base 10) system. The first steps in manipulating quantities begins with increasing (and then reducing) one value by another (addition and subtraction), before moving on to calculating the product of two values (multiplication) and its inverse, division. At this stage, children can also begin *handling* real-world objects through an understanding of shape and measurement. The four core operations form the basis for the further mathematical manipulation of numbers. For example, understanding fractions, long multiplication (and division), irrational numbers, rounding, place value and quotients. The complexity with which children represent and manipulate rational numbers requires a mixed approach to teaching, with the way learning takes place varying between children (Behr et al, 1983).

In all of the above numerical operations, being able to easily access and rapidly understand the underlying concepts and procedures requires ready computation of number facts - but we don't really mean computation. **Memorisation**. For a decimal system, that means multiplication up to 10x10 and addition up to 20 (because we often add two numbers together). But why memorise number facts? Some simple reasons:

1. **Access** – fluid mental agility with numbers makes their computation quicker and easier to undertake. Crucially, little cognitive effort is required to perform the calculation, allowing the child to focus on the aspect of numeracy that they are discovering. In literacy, this is akinto a good knowledge of vocabulary (and spelling) allowing effort to be devoted to the formulation of more expressive sentences.
2. **Confidence** – memorising number facts is not based upon any pre-defined talent or ability. Anyone can learn them and being shown this develops **confidence** in number manipulation, as well as instilling the belief that you can **do** maths. When learning new concepts, those children that are less successful are often at a disadvantage from slow computation, which may be a result of poor number fact knowledge. A **scaffold** of **knowledge** fosters a belief in their own ability and so positive feedback in their learning. For example, when children are presented with long multiplication, times tables knowledge is required to rapidly and accurately calculate partial sums and then use number bonds for the remainders.
3. **Patterns** – patterns in numbers become evident when viewing number facts (e.g. 11x table) and form the basis for calculating their values and then memorising them. This helps in understanding numerical patterns elsewhere in the curriculum.
4. **Estimation** – estimating (rather than computing) values is a crucial skill in all aspects of life (e.g. the approximate total of shopping) and times tables enable rapid mental calculation, allowing your conscious mind to focus upon the question at hand.
5. **Real World** – maths is used everywhere. Shopping, business, DIY, personal finance – numerical agility allows you to take responsibility for your actions from a position of understanding. For example,

- what is the cheapest unit cost (per 100g) of cheddar cheese in a supermarket?
- what are the best rates for the lowest *monthly* mortgage payment? How much lower will the *total* repayment be after increasing the monthly payment?
- how do builders use the 3-4-5 method to build right-angled corners?

Charlie Stripp (Director, National Centre for Excellence in the Teaching of Mathematics) outlined some of these points for the Times Education Supplement.

Of course, this doesn't explain why times table memorisation often goes to 12x when the decimal system dominates calculation within modern society. This is primarily historic, reflecting the pre-decimal monetary system that operated in the UK (£sd) that was base 12 and required dexterity when counting in twelves. Likewise, time is based upon multiples of 12 (particularly 60), angles work in factors and multiples of 360 and some imperial measures operate in multiples of 12 (e.g 12 inches in a foot). Learning the 11 and 12 times tables can also provide a measurable benefit for estimating values and introduces the first series of really interesting numerical patterns that forms the basis for pattern recognition in maths later on.

Some believe that these represent more niche applications and that the extra effort to memorise the 11 and 12 times tables is not an effective use of time (although our work suggests that the 12x table is not as difficult as they suggests). The reverse is that expanding memorisation to higher numbers would further assist computation, but at the obvious expense of an increasingly large quantity of number facts that are potentially more abstract. At some point a line must be drawn, after which you move to computation.

National Curriculum

For the UK National Curriculum, the first level of study (Key Stage 1) for numeracy specifies detailed programmes of study. These outline what learning should be achieved by each year of schooling. For number facts, these are:

Number Bonds
Year 1 (age 6): 1s to 10 (calculated), 1s to 20 (calculated)
Year 2 (age 7): 1s to 10, 1s to 20, 1s to 100 (calculated)

Times Tables
Year 1 (age 6): none
Year 2 (age 7): 2s, 5s and 10s
Year 3 (age 8): 3s, 4s and 8s
Year 4 (age 9): 6s, 7s, 9s, 11s, 12s

The focus on number facts is worthy of note as the Department for Education placed great emphasis on them when the 2014 curriculum was published. This reduced the existing requirement for 1-12x memorisation at age 11 to age 9 and proposed the introduction of specific electronic testing at age 11. Number facts have become high stakes for a schooling system based upon league tables, although these results may not be factored into actual league table calculations.

HOW we learn

Teachers adopt a range of different techniques for teaching number facts. These begin with the concepts of arithmetic and real numbers, becoming familiar with physically manipulating real-world objects and counting on and back. This naturally leads on to the introduction of the number line and being able to count in 1s before working with number bonds to 10 and 20.

With these first steps in addition and subtraction for real numbers embedded, children are then able to move on to counting on in quantities other than 1 and this introduces them to multiplication (and later division). Working with physical objects again kinaesthetically embeds the notion of how multiples operate. With this platform for multiplication established, what techniques exist to help children learn their times tables and what is the basis for this learning? Here we list some of the most common methods and broadly categorize how they address this challenge.

1. **Calculation**: this involves the actual manipulation of values or physical objects to actually compute the final value of a sum. Methods for achieving this include:

> **Fill-in-the-Square**: a 144-grid square (Figure 3) that requires the child to complete the product for each sum. Variations include focusing upon partially completed grids to calculate the squares, individual tables or random questions. *Strengths*: efficient, self-placed activity which is flexible in deployment. *Weaknesses*: unregulated, effort can be minimal and requires marking for feedback.

X	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Figure 3 144 Grid Answer Square

> **Mini Whiteboard**: interactive question and answer session where a child can practise writing and provide an immediate answer to a posed question. *Strengths*: flexibility in the question types that can be posed (e.g. times tables, counting on), allows instant feedback in fairly large groups. Relatively time efficient.
> *Weaknesses*: relatively few questions answered, mixed ability is difficult to manage and in moderate size groups children can copy one another.

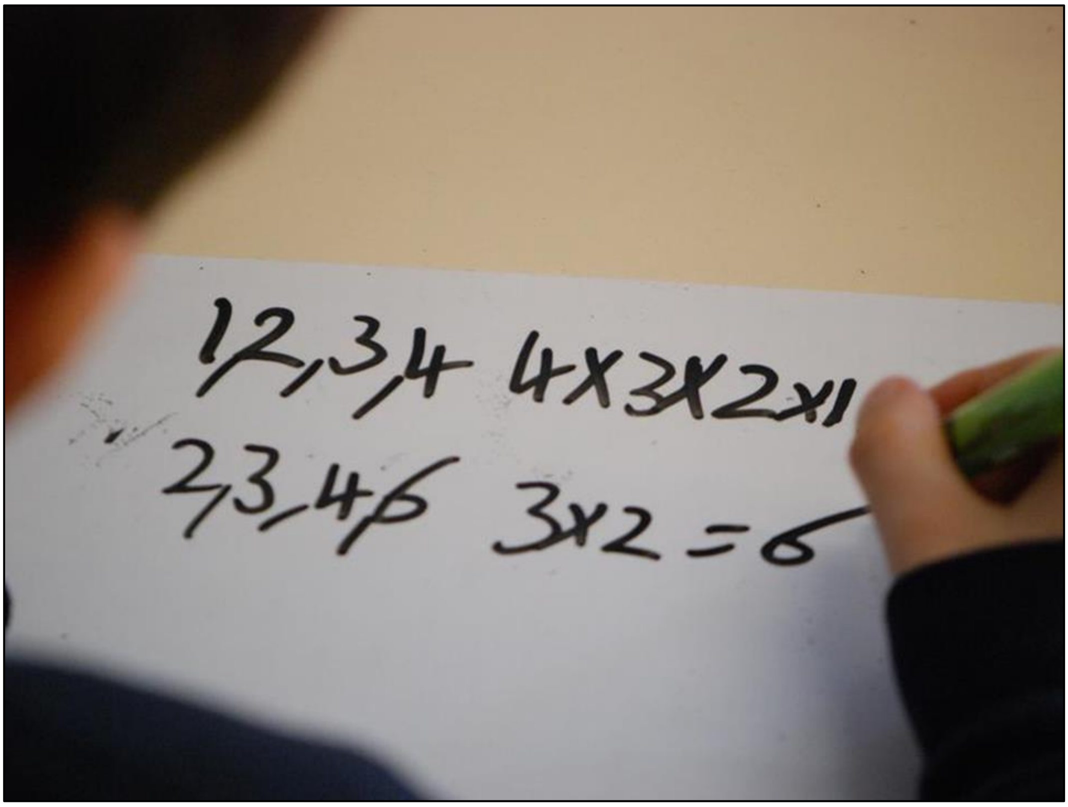

Figure 4 Whiteboard for answering number fact questions

2. **Mnemonics**: a range of techniques that tap into auditory or visual (long term implicit) memory in order to aid recall.

> **Chanting**: many adults will have strong memories of chanting their times tables in class, both forwards and backwards. This embeds the rhyme in memory and once you start reciting a times table it can then be recalled with ease. *Strengths*: embeds the "song" in implicit memory enabling ease of recall, whilst the children often enjoy the auditory learning.
> *Weaknesses*: the tables are only learnt in order and whilst it teaches the pattern of incrementing values (so providing knowledge to access the particular table) it doesn't allow instant recall of specific sums.

> **Metre Rule**: a long ruler (often a metre) is often used as it allows counting in multiples of up to 10. Used in conjunction with chanting, it enables a physical cue to the sound, whilst providing a rhythm acting like a metronome. *Strengths*: taps into physical cues allowing chanting to maintain a rhythm, whilst the finger moving along the rule provides visual reinforcement. *Weaknesses*: tables are learnt in order, making the additional 11x and 12x table harder to undertake. Only children near the front can see properly.

> **Rhymes**: simple rhymes or acronyms to help remember table sequences. These include:

"Knock knock knock on the door, eight times eight is sixty four"

"5, 6, 7, 8 for 7x8 = 56"

Strengths: allows fixing a fact in memory in a fun way, so enabling them to access practise.

Weaknesses: is not a replacement for implicit recall of number facts.

3. **Teaching Tricks**:

> **9s Knuckles**: many of the tables have repetitive sequences, however it can be difficult to memorise them. For the 9s, hold down the finger for a specific question (e.g. 4x9), which leaves 3 fingers to the left and 6 fingers to the right – 36 is the answer. This links to knowing that that the digits in the 9x table add up to 9. Or that the sum of a multiple of 11 can be inserted into the multiple: 36 x 11, means 3+6=9, giving 396 (although watch for carrying numbers and place value).

Strengths: an easy method of calculation that allows the 9x table (up to 10) to be rapidly attained.

Weaknesses: simply acts as a memory aid, but does not actually facilitate implicit storage and which may hinder further application.

> **Post-It Notes/Flashcards**: saturate your physical space with reminders, or cues, so that you can consciously bring "sticky" number facts back into your consciousness. This can include placing post-it notes in the front of books, on cupboard doors or the back of the toilet seat (!), as well as using flashcards to prompt for number fact recall.

Strengths: a visual reminder to consciously practise the retrieval of number facts.

Weaknesses: rather than assisting learning, notes can become a crutch that are relied upon, rather than a prompt for memorisation.

How we MIGHT learn?

Current teaching methods are firmly rooted in memorisation methods that are known to work, but crucially need to be deployed as either group teaching or homework. This requires the child to engage with their learning and actively participate in acquiring number fact knowledge. It should come as no surprise that this doesn't often happen given the limited time that can be devoted to number fact acquisition. However, the **extent** of the problem has not been well understood and until baseline measurement is undertaken, many teachers will underestimate the lack of progress in this area and have assumptions that their pupils are acquiring number facts.

For memorisation to be effective, any teaching techniques must be able to effectively move number facts from **explicit** to **implicit** memory. Based upon Section 2, there are six techniques that can be applied. Note that this information draws heavily upon research summarised in "Brain Rules" (Medina, 2014) and "Make It Stick" (Brown et al, 2014) (see the summary article in Mind by Dunlosky et al, 2013).

1. **Massed Practise**: as the maxim goes *"repeat to remember"*. Repeating something over and over, be it number facts, spellings or your badminton backhand volley, will lead to long term memory storage. Practise, practise and more practise. There is no secret about putting in the time and just keeping practising. Malcom Gladwell In "Outliers" (2009) notes that research suggests 10,000 hours of practise is required to become world leading (as a pianist, chess grand master or tennis player).

   Take-Away: practise often, although remember that the memory is **not** a muscle. Practising memorising the capitals of Europe will **not** make your number fact knowledge better!
2. **Spaced Repetition**: Figure 2 showed the dramatic effect of loss of knowledge through the forgetting curve. Ebbinghaus experimented with a technique called spaced repetition and saw dramatic improvements in remembering. By repeatedly exposing his subjects to the same knowledge, at regular intervals, he was able to effectively halt the loss of information. As you become more secure in memorisation you can increase the period of time between re-exposure to that information.

   Take-Away: regularly refresh yourself with the same information in order to remember it. For number facts that means **regularly** practising the times tables and number bonds (which means a minimum of three times per week).

3. **Purposeful Practise**: a phrase coined in "Bounce" (Syed, 2011), purposeful practise is about working hard, at the edge of your ability, trying with all your effort. Purposeful means being engaged, wanting to practise to the full extent of your ability and being prepared to fail, in order to achieve beyond your current capabilities.

   Take-Away: going harder, at the edge of ability. If you can't access the 7x table, practise them in order to get more familiar with the number facts. If you can calculate all your times tables, press hard on how *quickly* you can answer them in order to push for implicit memorisation. This will allow you to move from calculation to recall.

4. **Feedback**: an understanding of your ability is crucial to gauging where the gaps in your knowledge lie, assess where you are going wrong and so devise strategies to improve. Feedback is important for children to understand where and why they make mistakes, whilst it allows teachers to identify individual and group weaknesses, targeting interventions in these areas.

   Take-Away: regular feedback on what is going right and what is going wrong must be provided. This can be immediate whilst learning, immediately post-learning and then summary feedback on a weekly basis so that overall progress can be gauged.

5. **Calibration**: this follows directly on from feedback. In order to accurately gauge performance, metrics that are based upon standardised testing are needed. *Low stakes* testing is a critical part to the process enabling a detailed understanding of an individual's progress.

   Take-Away: build testing into any activity, but make it part of the learning process so that it is a regular part of the everyday. Additionally, this can inform senior leaders of achievement, guide governance and be used as evidence for school inspection. It is common for both learners and teachers to *over-estimate* the acquisition of knowledge and how long it takes to deeply embed it. For teachers who *know* their number facts, it is often hard to understand why a child hasn't learnt this information sooner, a misunderstanding known as the "curse of knowledge." We underestimate the difficulty in acquiring new knowledge by heart.

6. **Interleaving**: overall learning is significantly improved by *interleaving* different topics. This *feels* counter-intuitive because when we practise one thing to a level of mastery, the gains appear substantial. However when learning is interleaved, the **overall gains** are higher at the end of the training period.

Take-Away: interleave different times tables, times tables and number bonds or times tables and spellings!

All of the above techniques are proven to improve learning and the memorisation of knowledge, moving information from explicit to implicit memory. Can we design number fact learning to take advantage of these substantial gains? In the next section, we present a tested model demonstrating the level of impact on learning that can be achieved.

Case Study: SEN

By Ellie Field, Class Teacher, Caddington Village School

Ben joined one of our Year 3 (age 8) classes midyear, and settled in quickly – a sociable, charming and bright boy. I marked his books, saw him contributing in class using the mini whiteboards and had him mentally pegged as a capable child, coping well with the demands of the curriculum. I had no concerns and waited for his Year 2 results to come through from his previous school.

We use Number.fy routinely for learning number facts and often work with the children in small groups. Shortly after Ben arrived I was working with a group of children and set them a number of times tables to complete. Afterwards, I reviewed the results, keen to see what the data showed regarding their knowledge and progress. It was very easy to spot that one child who was not doing very well, in fact it was obvious that he did not even know his 1x table… the child was Ben.

Upon investigating further, it transpired that Ben had developed a brilliant coping strategy – he was adept at copying, really adept! His Year 2 test results were above average for reading (3a in old levels) and a high average for maths (2a) – so he had fooled his previous school too! His Mother was much relieved as she had always felt that something was not quite right and so was keen to co-operate with the Special Educational Needs referral that was processed immediately by our SENDCo.

Number.fy is not only part of our daily class routine but invaluable for children like Ben – there was no place to hide as it was just him and a non-threatening mobile device. His failure became a success story as he subsequently received the intervention and support he needed.

Optimal Number Fact Learning

Given what we know about the importance of learning number facts, the requirements of national curriculums, the nature of memory, remembering, forgetting and strategies for teaching, what might an optimal number fact learning system actually look like? In this section we present one such strategy that integrates together many of the known gains that can be made in memorisation.

What do we need?

For many of the strategies identified earlier, a computer based system is ideally suited. In particular, massed practise, feedback, calibration and interleaving, and can help with spaced repetition and purposeful practise. But, these capabilities have been around for many years in the form of PC based applications. The challenge for interweaving the benefits of augmenting classroom learning with electronic devices is that it must be seamless for the teacher and the child. It must be as simple as **pick the device up and just play**.

It's worth remembering that **computers don't build skills, teachers do** – Information Technology is simply another way of assisting learning and it must integrate in with the way learning takes place in and out of the classroom. The teacher-pupil relationship is critical to the learning journey and any "teaching tools" should amplify this.

On this basis, we have made some core assumptions about deployment:

1. **Device**: this **must** be portable for use anywhere, ideally with one device per child and in a small form factor so that it is physically easy to handle for all ages of children.
   Solution: a mobile phone or small tablet (7") device.
2. **Login**: any computer based system requires the child to login in order to identify them. This should be quick and accessible to children as young as age 5. *Solution*: a visual/graphical login that avoids keyboards/words.

3. **Recording**: all questions to be logged and stored centrally. *Solution*: logging each question answered and returning the answer to a centralised server.
4. **Massed Practise**: rapid and dense question delivery that allows a large number of questions to be answered in a short period of time.
   Solution: short, rapid fire, questions to maximise the efficient use of time in the classroom.
5. **Feedback**: recording all questions answered by pupils, followed by detailed feedback.
   Solution: immediate feedback on a question-by-question basis to show success/failure, summary feedback at the end of a set of questions and then centralised feedback to the teacher on all pupils to allow rapid assessment of progress.
6. **Testing**: a standardised assessment that allows direct comparative measurement of the progress of individuals.
   Solution: a short set of randomised questions that form the basis for baseline testing and monitoring of progress.
7. **Interleaving**: intermixing a range of different question types. *Solution*: a mix of different questions such as in order, out of order, individual times tables or randomised questions.

The above design constraints formed the basis for the Flurrish Number.fy app which implements the following features:

> operates on a handheld device
> can be integrated into classroom practice
> uses a graphical login system
> allows the selection of a range of number bond and times table "games"
> can interleave a range of different question types
> provides question and "game" feedback to the child
> incorporates a randomised testing game for monitoring progress
> allows a high density of practise – children at the early stages of 12x acquisition can answer 20 questions in less than 2 minutes. Confident learners can answer in less than 1 minute
> uses a centralised server to store all results
> presents a website to the teacher with rapid, graphical, "at a glance" results by class and individual pupil

Number fact learning at Caddington Village School

Number facts are introduced, in line with the UK National Curriculum, in Year 1 with a strong focus at Year 2 on acquiring a firm foundation in times tables that can then be sustained with regular "top-up" practise in later years. Extra attention is applied in Year 4 (to ensure that implicit memorisation is maintained) and Year 6 when national standardised tests are undertaken by all children.

Figure 5 Using Number.fy at Caddington Village School

Year 1 (age 5-6) : introduced at the beginning of year 1, "playing" number bonds is simply a part of routine learning, a table activity that takes 10-15 minutes to complete. This enables children to become familiar with technology at an early age. Answers are logged to monitor progress and can help identify those that might have special educational needs, or would benefit from early intervention

Year 2 (age 6-7): times tables are formally introduced and it is here that **daily** practise for 10-15 minutes begins. Before they can be committed to implicit memory, pupils need to be able to *access* the times table answers through chanting, counting on, rhyming, post-it notes and lists. Massed practise through Number.fy is simply one of a range of learning strategies to engage pupils. Times table learning starts with the 2s, 5s, and 10s and at the end of the first term most children should be secure in accessing this knowledge and practising it. Term 2 progresses to practising the 3s, 4s, 6s and 11s, before finishing in term 3 with the 7s, 8s, 9s and 12s. By the end of Year 2 experience shows that over half the cohort of children should have times table knowledge deeply embedded with instant recall of number facts – and will have answered ~10,000 each! Similar to literacy (and a "pen license"), children receive certificates as they lear their times tables, with a final *Times Table Master* at the end.

Year 3 (age 7–8): regular practise 2–3 times a week is now undertaken to ensure that memorisation is maintained. For those that hadn't acquired full 12x table knowledge, Year 3 is about consolidation and focusing upon completing that knowledge. All children should ideally be able to answer 20 random questions within 1 minute by the end of the year.

Year 4 (age 8–9): renewed focus now occurs with greater emphasis upon practise, and to ensure **all** pupils reach the National Curriculum target of secure knowledge. Practise is daily with all pupils focusing upon completing the interleaved random 20 questions to maximise learning benefit and answering a minimum of 100 per day. This remains a 10–15 minute starter activity.

Purposeful practise is stressed and, as knowledge is secure, children are encouraged to reduce the amount of time taken to answer 20 questions whilst maintaining as many correct answers as possible. Any child who achieves under 40 seconds has become fluent in their knowledge. The very best students will answer in under 25 seconds.

Year 5 (age 9–10): similar to Year 3, practise is 2–3 times per week with a target of maintaining the same levels of achievement from Year 4. There remains a focus upon purposeful practise by reducing answer times whilst maintaining accuracy.

Year 6 (age 10–11): with national standardised tests at the end of year 6, there is a strong focus across all academic subjects that are tested to ensure that all pupils meet the minimum benchmark standards. Times tables are important for pupil confidence and accessing higher levels of the numeracy curriculum – whilst not a core focus for testing, practise is 2–3 times a week, with careful monitoring to ensure all pupils are secure in the full 12x tables knowledge.

Evidence: But does it work?

For any learning intervention, it is important that progress against benchmarked standards are monitored so that its success or failure can be assessed. Caddington Village School therefore ran a year long trial for a Year 2 class of 23 pupils, switching to a teacher led maths curriculum centered around using Number.fy for practise, monitoring progress through the logged results and then focusing upon the individual needs of each learner. The results were nothing short of astonishing.

To demonstrate the outcomes of the trial, three graphs are presented below. These show how many individual times table questions were answered by the class, playing daily for a period of approximately 15 minutes:

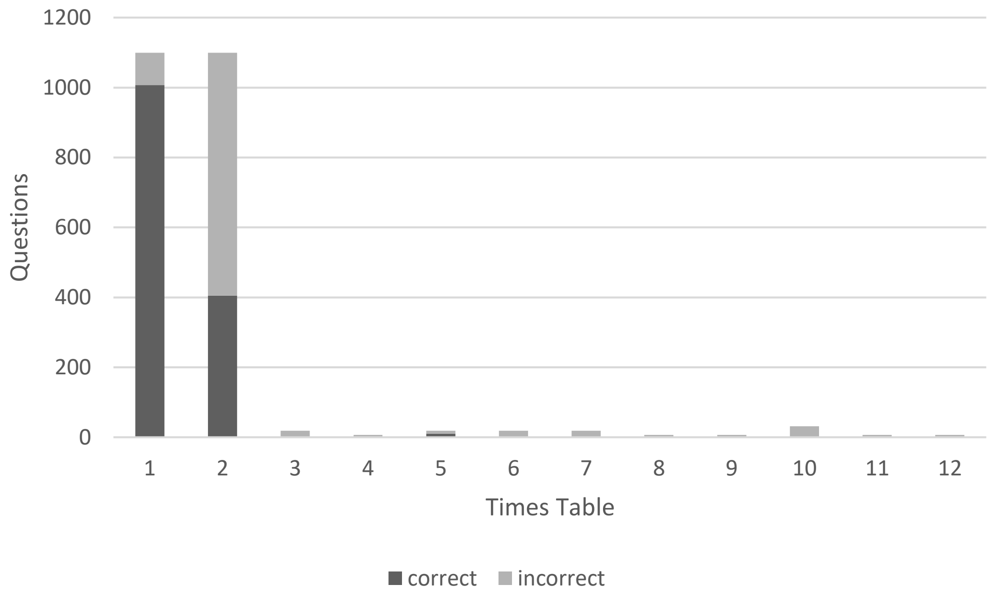

Figure 6: this shows the first full week of school with 2340 questions answered, 61% of which were correct with a focus upon the 1s and 2s.

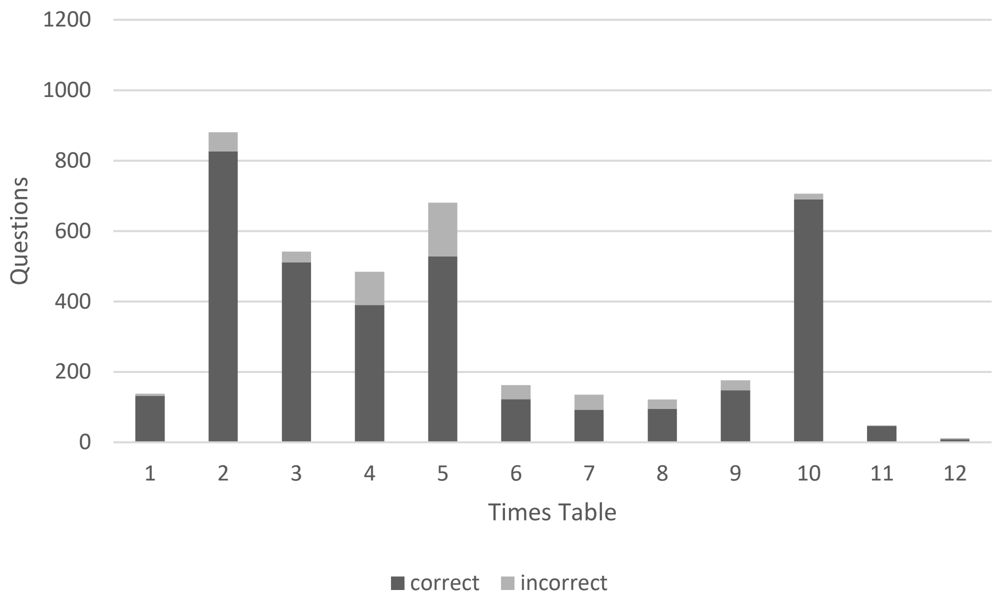

Figure 7: this shows a full week at the end of Term 1 and here we see a dramatic improvement – 4088 questions answered, 88% correct and now 1s, 2s, 4s, 5s and 10s, with a smattering of the other times tables (the class had answered 51,000 questions by this point!).

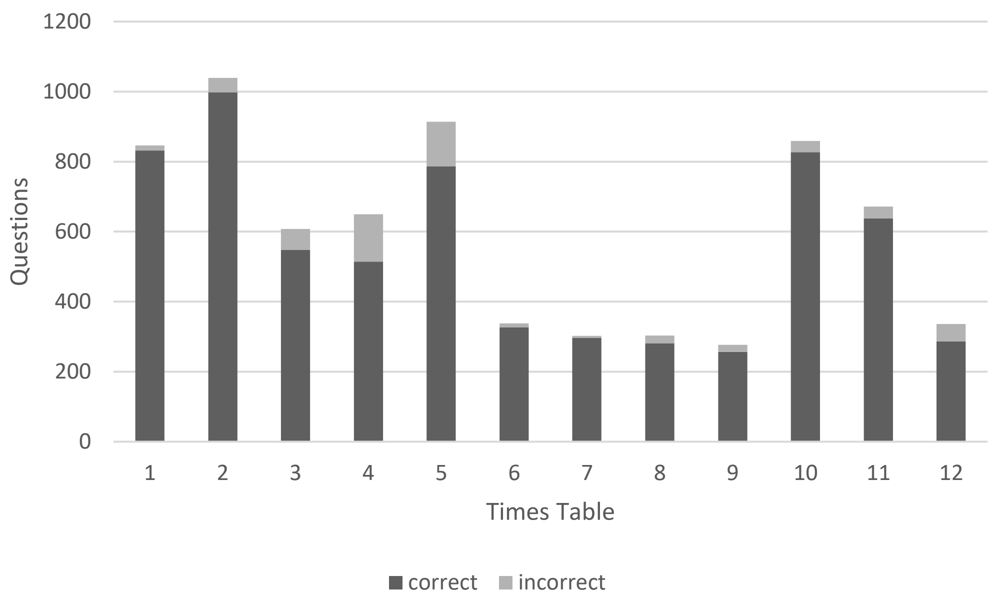

Figure 8: the final graph shows a full week in the middle of Term 3 with 7144 questions answered, 92% correct and now a good number of 6s, 7s, 8s, 9s and 12s (170,000 questions answered by this point!).

Just as a reminder, the National Curriculum requires Year 2 children to know their 2s, 5s and 10 times tables. In terms of individual successes – 65% of the class consistently knew all of their 1-12x tables.

These results show dramatic **impact**. There is clearly increased knowledge, increased accuracy and increased speed resulting in a dramatic increase in achievement and a massive boost in pupil confidence, which will provide the perfect springboard for future mathematical learning.

(We are grateful to Richard Kingham and all the children in 2K for their hard work and enthusiasm over the year).

Case Study: Building Confidence

By Joe Teague and Merrick Davies, Maths Teacher, Caddington Village School

Number.fy is used on a daily basis in our school and, across all our year groups. It has underpinned some key principles for building confidence in pupils.

Praise: meaningful praise is highly effective for learners and the immediate feedback provided by Number.fy – for individual pupils and for the teacher - completes this loop in learning. When Number.fy is played on smart devices, the chirrup that is heard when an answer is correct is intrinsically pleasing and a whole classroom of pupils playing really gives us 'the sound of maths' and the feeling of success.

'Well done Gemma, you have made progress and now know all of your 6 times tables.' Mr Teague

Build resilience: getting things wrong happens to everyone and is good for learning. Many children dislike the perception of "failure" but it's important to see it as part of learning – teachers can build a culture of showing mistakes and learning from them. By going through the number fact answers on a big screen the teacher can drill down to see common mistakes and pupils know that it's OK to get the wrong answer and learn from that. Haptic feedback provided on the smart devices when a wrong answer is input is a private, gentle and tangible reminder for pupils – this feature is particularly good for very young children or children with SEND. Pupils need to be encouraged to look at the feedback screen following each game and learn from their mistakes, rather than just hitting the 'play again' button. The cycle of try, review, learn, try again is very powerful and should be encouraged –it is something that children have to learn - as they often see it as a waste of time.

'When I get one wrong, I just carry on and try and learn it for next time.' Evelyn Year 2

Growth Mindset: many children find maths difficult and quickly lose motivation, not realising that they are learning when they get things wrong. Number.fy provides clear evidence of progress, which enables children to see, and believe, that they can learn maths. They understand that practise is the key to achieving number fact knowledge and can then transfer that belief to other areas of learning.

'I lie in my bed and look at my times tables poster so that I can get more questions right.' Amir Year 2

Independent Learning: one child with one device gives the best results - if you have a half class set of devices then play two lots of games rather than put pupils in pairs. Pupils need to get a true measure of what they know and don't know. However, teachers should encourage discussion of how to remember – to share tricks and tips as this can generate some powerful discussions in the classroom.

Fun: playing Number.fy is fun! It is seen as a game and children love to get better at playing it. Even those that know their times tables want to get faster and a surprising number are incredibly quick. Number.fy is also not bogged down with unnecessary features – the rewards are quite simple, getting better and faster at times tables and number bonds.

'My fastest time is currently 17.38 seconds for the random 20s - I love being good at my times tables!' Ella Year 6

Summary

In this whitepaper we have outlined the rationale for learning number fact knowledge, the principle components of memory and how we forget, why we learn number facts, how we currently undertake this and what an optimised method for learning might look like. We conclude by summarising the development of a practical implementation of this, how Caddington Village School have applied this across their year groups and the results of a year-long trial.

Some of the **BIG** take-aways that are worth repeating:

> achievement in numeracy is a key aspect of modern life - you need to be numerate for exam performance, career development and, ultimately, life chances. Ability in numeracy increases self-esteem and confidence and can have a positive impact across other aspects of life

> memory controls encoding, storage and retrieval of information

> short term memory can only store small lists of items for short periods of time

> long term memory can be explicit (conscious abstract knowledge or personal memories) or implicit (unconscious information)

> memorisation (rote learning) of number facts provides essential scaffolding for the basics of numeracy and allows children to rapidly and confidently build upon a secure foundation

> current classroom practice is founded upon good learning principles but lacks the volume of massed practice and high quality feedback to be effective for all pupils

> the Number.fy app is targeted to meet the "pick up and play" requirements of the 21st century classroom by being: handheld, easy to login, high density of practise, rapid feedback, interleaved questions, standardised testing and centralised recording and marking of all results

> a 10-15 minute starter activity can easily generate 40+ questions per pupil. For a class of 30, that's 6,000 questions per week. Or 228,000 questions per year!

> a year-long trial of 23 age 7 children (Year 2) saw them answer 170,000 questions with 65% of them secure in the 1-12x table knowledge by the end

In the book "Make It Stick" (Brown et al, 2014) the authors stress the importance of memorisation and testing to help children form a strong foundation in their learning. They undertook a year-long controlled study at Columbia Middle School with impressive results. The strength of this approach is perfectly exemplified by the thoughts of the Headteacher:

"What I've really gained a comfort level with is this: for kids to be able to evaluate, synthesize and apply a concept in different settings they're going to be much more efficient at getting there when they have the base of knowledge and the retention, so they're not wasting time trying to go back and figure out what that word might mean or what that concept was about. It allows them to go to a higher level."
Roger Chamberlain, Columbia Middle School

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